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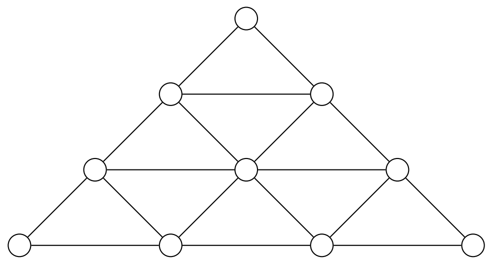
Advanced Algorithms

Exercise Sheet 2

Submission: Monday, April 21, 2025, at 11:59 am.
This exercise will be discussed on Wednesday, April 30, 2025.

Exercise 2.1 (Algorithm for maximum non-bipartite matching) (10 Points)

Compute a maximum matching of the graph given below. Choose the shortest paths in D_M such that you apply at least 3 times the 'shrink' operation of the algorithm.



Exercise 2.2 (Simple (approximation) algorithm for matchings) (6 Points)

Consider the following simple algorithm to compute a matching in a given graph G. We fix some $k \in \mathbb{N}$, $k \geq 1$.

1. Let M be the current matching.
2. If there is a set of $k' \leq k$ edges $M' \subseteq M$ and a set of $k' + 1$ edges $F \subseteq E \setminus M$ such that $(M \setminus M') \cup F$ is a matching, update M to $(M \setminus M') \cup F$. Repeat.

Show the following:

a) The running time of the algorithm is $O(|E|^{2(k+1)})$.

b) Let M be the matching output by the algorithm and let M^* be a maximum matching in G. Show that $|M| \geq \frac{k+1}{k+2}|M^*|$

Exercise 2.3 (Slither game) (4 Points)

Consider the game *Slither* by W. Anderson. Given an undirected, simple connected graph $G = (V, E)$, two players choose in turns an edge e with the following rule: e was not yet chosen and the set of up to now chosen edges (including e) represents a simple path. The player who cannot choose an edge according to the rules loses.

Show: If G contains a perfect matching, then there is winning-strategy for the first player.

The following exercises are no homework and shall be discussed in presence.

Exercise 2.4 (Covering sets of vertices)

Let G be a bipartite graph with bipartition $V(G) = L \cup R$. Suppose that $S \subseteq L$ and $T \subseteq R$, and there is a matching covering S and a matching covering T. Prove that there is a matching covering $S \cup T$.

Exercise 2.5 (Unmatchable edges)

Given an undirected graph G, an edge is called *unmatchable* if it is not contained in any perfect matching. How can one determine the set of unmatchable edges in $O(|E| \cdot |V|^2)$ time? Hint: First determine a perfect matching in G. Then determine for each vertex v the set of unmatchable edges incident to v.

Exercise 2.6 (Edge Cover Problem)

In the Edge Cover Problem we are given a graph $G = (V, E)$ and the task is to compute a minimum number of edges $X \subseteq E$ such that each vertex $v \in V$ is incident to at least one edge in X, i.e., for each $v \in V$ there exists some $u \in V$ such that $uv \in X$.
Give an algorithm that computes in polynomial time an optimum solution to the Edge Cover Problem and prove its correctness and running-time.