University of Bremen

FB3: Mathematik/Informatik

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Advanced Algorithms

Exercise Sheet 5

Submission: Monday, May 19, 2025, at 11:59 am.
This exercise will be discussed on Wednesday, May 21, 2025.

Exercise 5.1 (Ford-Fulkerson with thickest paths) (10 Points)

Let $\mathcal{N}$ be an integer s-t-network and f be a feasible flow in $\mathcal{N}$. A *thickest* s-t-path in $\mathcal{N}_f$ is an s-t-path with a maximal bottleneck capacity.

(a) Prove that the number of augmentations needed by the variant of the Ford-Fulkerson algorithm that always augments along a thickest s-t-path in $\mathcal{N}_f$ for a network $\mathcal{N}$ with n vertices, m edges, and integer capacities in $\{0, 1, \dots, C\}$ is in $O(m \log(nC))$.

Hint: use Exercise 4.3 and the fact that $(1 - \frac{1}{x})^x \leq \frac{1}{e}$ *for all* $x > 1$.

(b) Describe an efficient algorithm to find a thickest s-t-path in $\mathcal{N}_f$, and analyze its running time.

Hint: recall Dijkstra's algorithm.

Exercise 5.2 (Blocking vs Maximum) (5 Points)

Let $\mathcal{N}$ be an s-t-network. Prove or disprove the following statements.

(a) Every maximum flow in $\mathcal{N}$ is also a blocking flow in $\mathcal{N}$.

(b) Every blocking flow in $\mathcal{N}$ is also a maximum flow in $\mathcal{N}$.

Exercise 5.3 (Number of Iterations of Dinitz Algorithm) (5 Points)

For every natural number k, describe an s-t-network on which Dinitz algorithm needs at least k iterations, i.e., computes at least k successive blocking flows. Explain why at least k iterations are needed.