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Advanced Algorithms

Exercise Sheet 7

Submission: Monday, June 2 at 11:59 am.
This exercise will be discussed Wednesday, June 4

Exercise 7.1 (Modeling as Min-Cost Flow) (10 Points)

Consider the following problem. A car manufacturer has r factories $F_1, \ldots, F_r$ to produce s different car models $M_1, \ldots, M_s$. For $i \in [r]$ and $j \in [s]$, it is known whether factory F_i can produce model M_j and, if so, at what unit cost $\alpha_{i,j}$. Furthermore, factory F_i for $i \in [r]$ can produce at most β_i cars per month, of which at most γ_{ij} can be of model M_j, $j \in [s]$. The cars are sold by t car dealers $H_1, \ldots, H_t$. For $i \in [r]$ and $k \in [t]$, it costs $\delta_{i,k}$ to transport a vehicle from F_i to H_k. Additionally, dealer H_k, for $k \in [t]$, sells $\theta_{j,k}$ cars of model $M_j \in [s]$ per month. The goal is to plan the production and transportation of cars such that the total cost is minimized.

Model the above optimization problem as a minimum-cost flow problem. Specify the set of nodes and edges, as well as the edge capacities, costs, and balances. Argue why your modeling solves the given production problem.

Exercise 7.2 (Decomposition into Cycles) (10 Points)

Prove the following lemma from the lecture:

Let f be a circulation in a network $\mathcal{N} = (V, E, c, b, p)$. Then, there exist flows $f_1, \ldots, f_k$ with $k \leq m$ such that

(i) $f = f_1 + \ldots + f_k$,

(ii) f_i is a feasible circulation in $\mathcal{N}$ for all $i \in [k]$, and

(iii) f_i takes positive values only on edges of a cycle C_i in $\mathcal{N}$ for all $i \in [k]$.