Universität Bremen

M.Sc. Neurosciences

Module MN-F3: Theoretical Neuroscience

Exercises #1 / Tutorial on 27.11.2025

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1 Understanding spikes

a) What is a firing rate?

b) What physical unit does a firing rate, its variance, and standard deviation have?

c) Consider an experiment in which you record multiple trials from a neuron in an impatient animal. Which trade-off do you have to consider if you want to estimate a time-varying rate as best as possible?

d) If you are able to obtain only one trial, how can you nevertheless estimate a (continuous) firing rate even for times where you did not observe a spike? Which downside does your method have?

2 Computing with spikes

In theoretical neuroscience, it is convenient to express spikes with a δ-function (or, more precisely, δ-distribution). You can compute with δ-distributions using the following rules:

$$\int_a^b h(t)\delta(t-t')dt = \begin{cases} h(t'), & \text{if } a < t' < b. \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

$$\delta(ax) = \frac{\delta(x)}{|a|} \quad (2)$$

a) A spike train is given by $\rho(t) = \sum_{i=1}^{n} \delta(t-t_i)$. Compute the total number of spikes by integrating over ρ over the whole time axis.

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b) You have recorded a (discrete) spike train $\rho(t) = \sum_{i=1}^{n} \delta(t - t_i)$ and want to try two different smoothing kernels $a(\tau)$ and $b(\tau)$ for estimating a nice, continuous firing rate:

$$a(\tau) = \begin{cases} a_0 & \text{if } -\tau_0 < \tau < +\tau_0. \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

$$b(\tau) = \begin{cases} -b_0|\tau| + b_1 & \text{if } -\tau_0 < \tau < +\tau_0. \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

- Sketch the two kernels.
- How do you have to choose the (positive) parameters a_0, b_0, b_1 for the kernel to be well-defined (see script for conditions!)?
- Show analytically that the spike count is conserved if you apply the appropriately normalized kernels to the spike train.

3 EXPERTS: Properties of the Poisson statistics

Poisson statistics has certain properties which make some computations for analyzing neural data easy. Here we will explore two of these:

a) If you have two independent Poisson processes with parameters λ_1 and λ_2, show that the spike count $k = k_1 + k_2$ of the combined observations has the same statistics as a Poisson process with parameter $\lambda = \lambda_1 + \lambda_2$.

b) By starting from a Bernoulli process and letting the bin size of δt go to zero, show that the probability for observing an inter-spike interval of size T follows an exponential distribution.