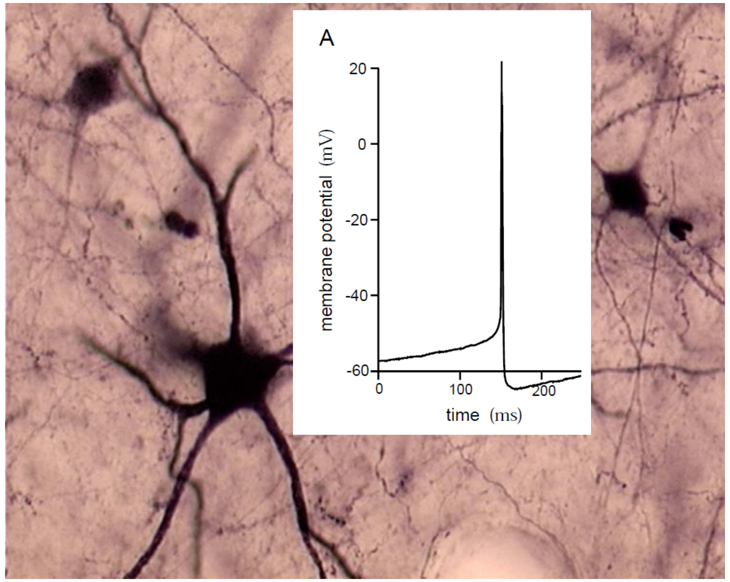
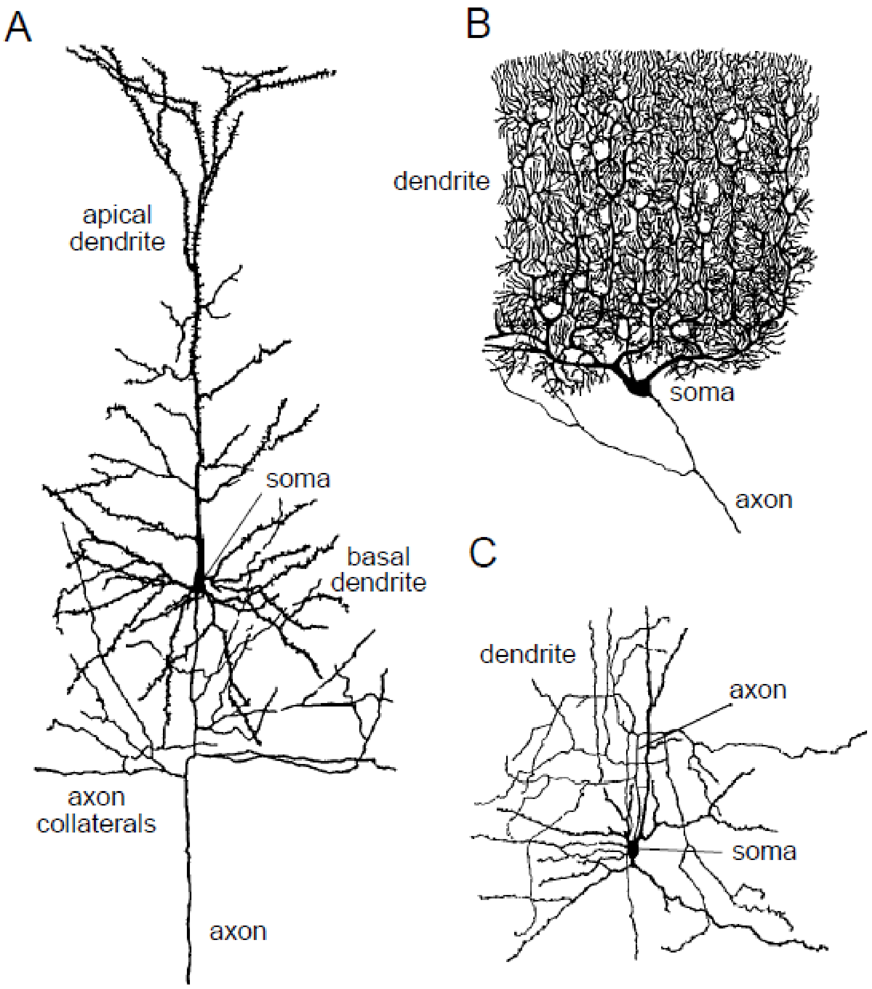
Lecture 1:

Spikes & Spike Statistics

Communication between Neurons

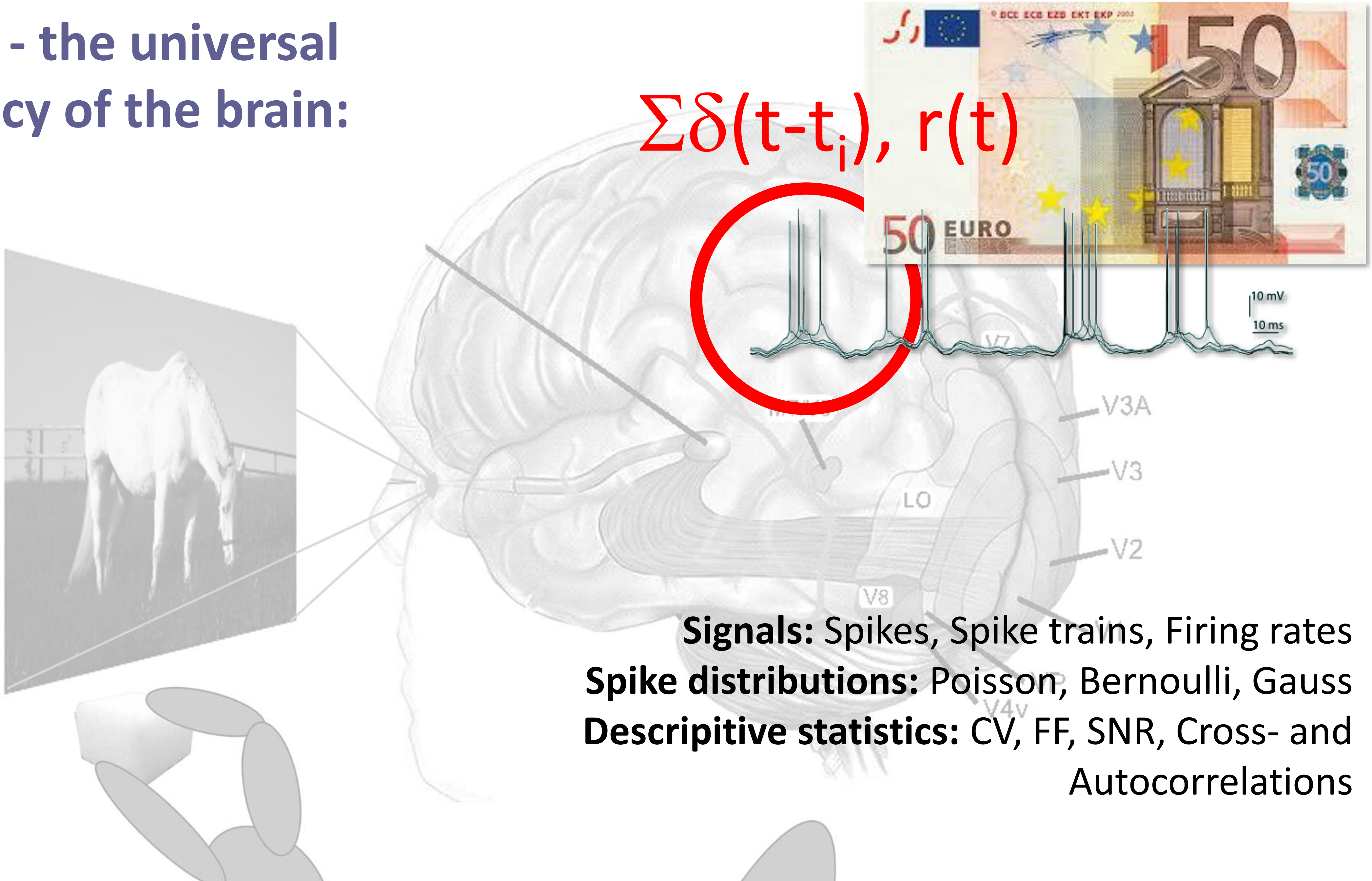
Most neurons communicate through the exchange of ***action potentials...***





Spikes - the universal currency of the brain:

$\Sigma\delta(t-t_i)$, $r(t)$



Signals: Spikes, Spike trains, Firing rates

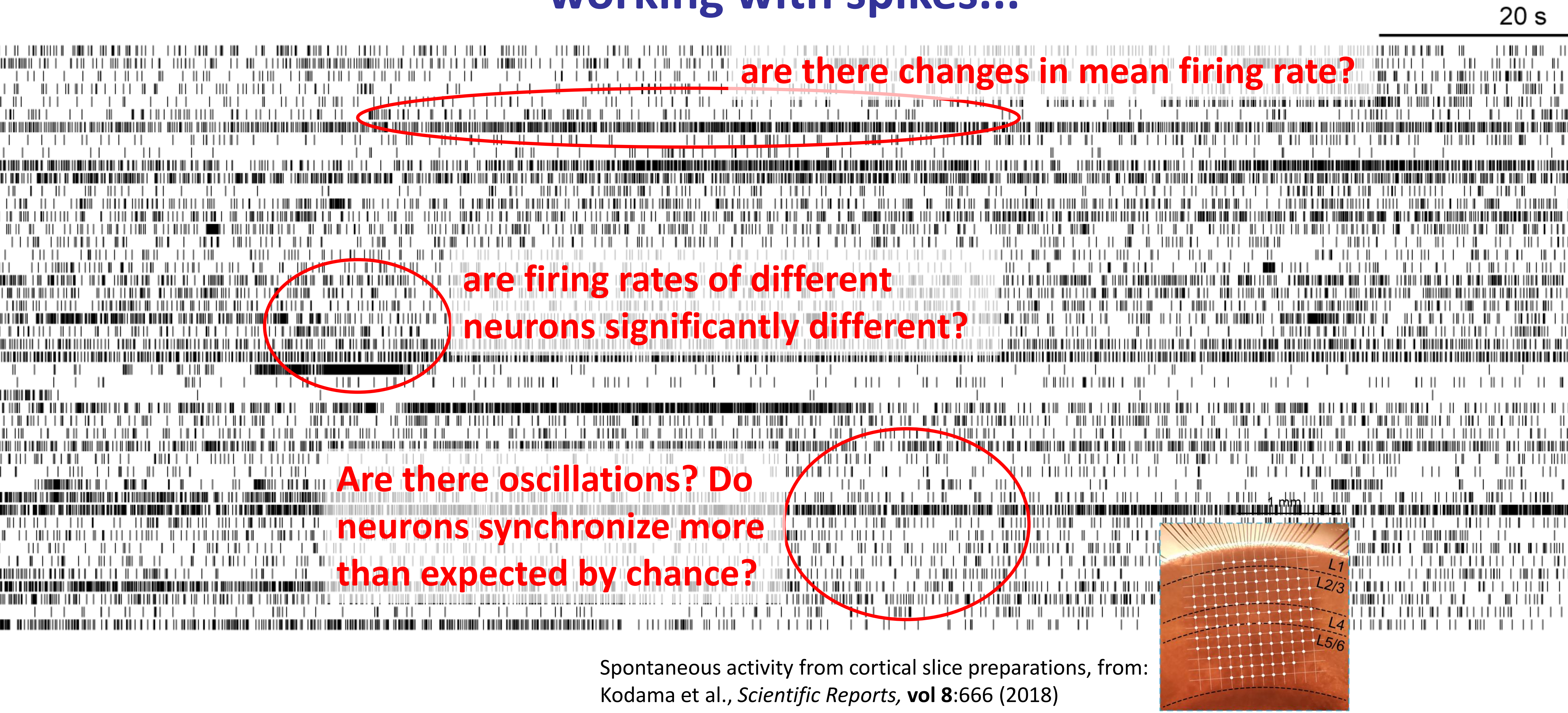
Spike distributions: Poisson, Bernoulli, Gauss

Descripitive statistics: CV, FF, SNR, Cross- and Autocorrelations

We also have to care about spike statistics!

- for **analyzing data**:
  you have to cope with noise, statistical tests
- for **understanding encoding**:
  you have to know statistics to draw meaningful conclusions
- for **decoding from the brain**:
  you do better if you know statistics

Some fundamental problems when working with spikes...



Spontaneous activity from cortical slice preparations, from: Kodama et al., *Scientific Reports,* **vol 8**:666 (2018)

Whiteboard!

- discretized representation
- continuous representation (delta fct)
- rules for computing with delta function
  (motivate heuristically, notation mathematically)
- spike trains as delta-fcts (count vs instantaneous)
- trade-off time vs. #observations

Experts Info: The ‚delta-distribution' – a convenient tool for computing with spikes in continuous time

The delta-'function' is a convenient tool for representing spikes – if a spike occurs at time t_{sp}, just write $\delta(t-t_{sp})$. For computing with δ's, you just have to know:

$$\int_a^b h(x)\delta(x-x')dx = h(x')$$

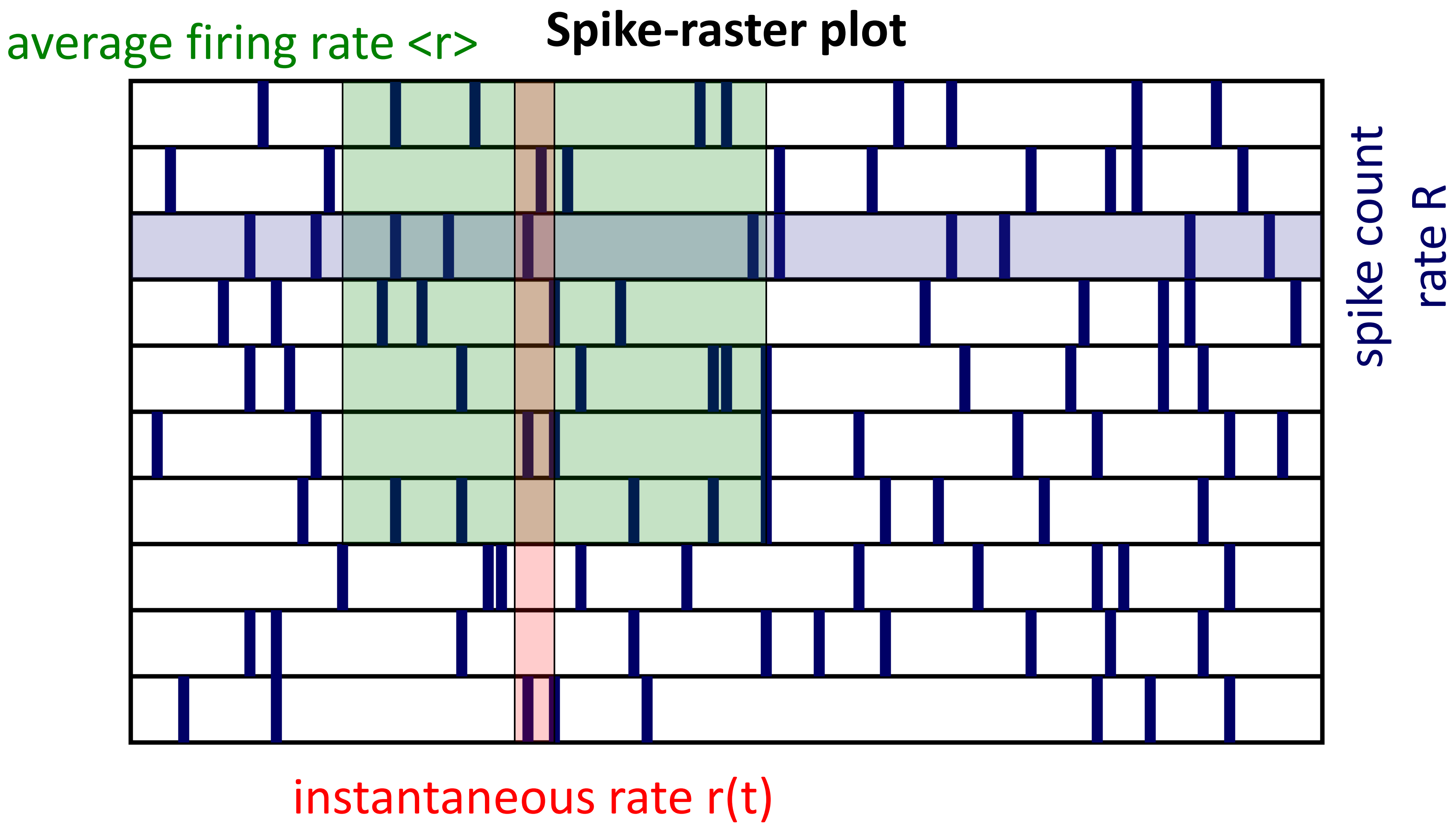
if $a < x' < b$ (*0* otherwise)

$$\delta(ax) = \frac{\delta(x)}{|a|}$$

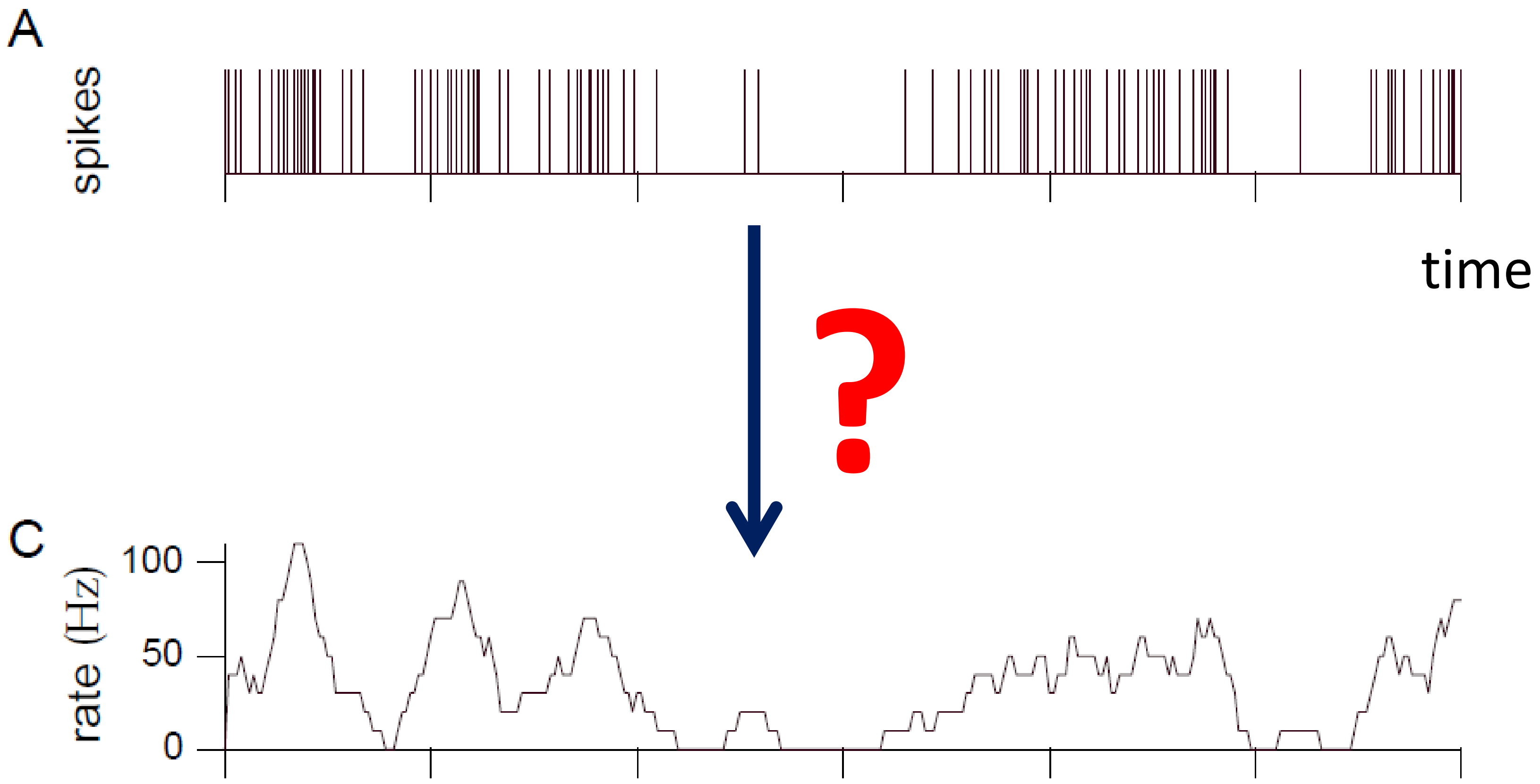
$$\int_a^b \delta(x-x')dx = 1$$

if $a < x' < b$ (*0* otherwise)

Rates from counting spikes - tradeoff between temporal resolution and precision



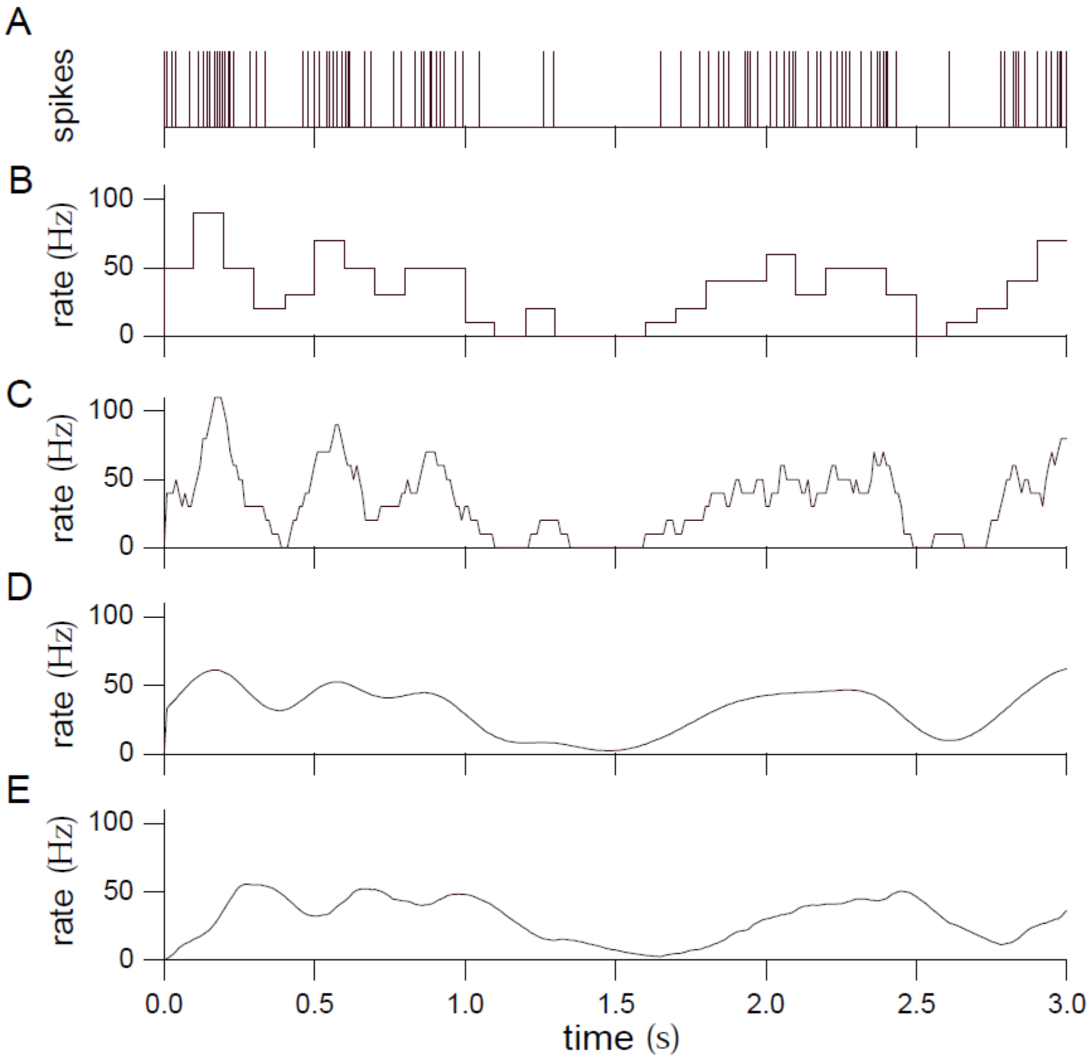
How do we estimate a firing rate?



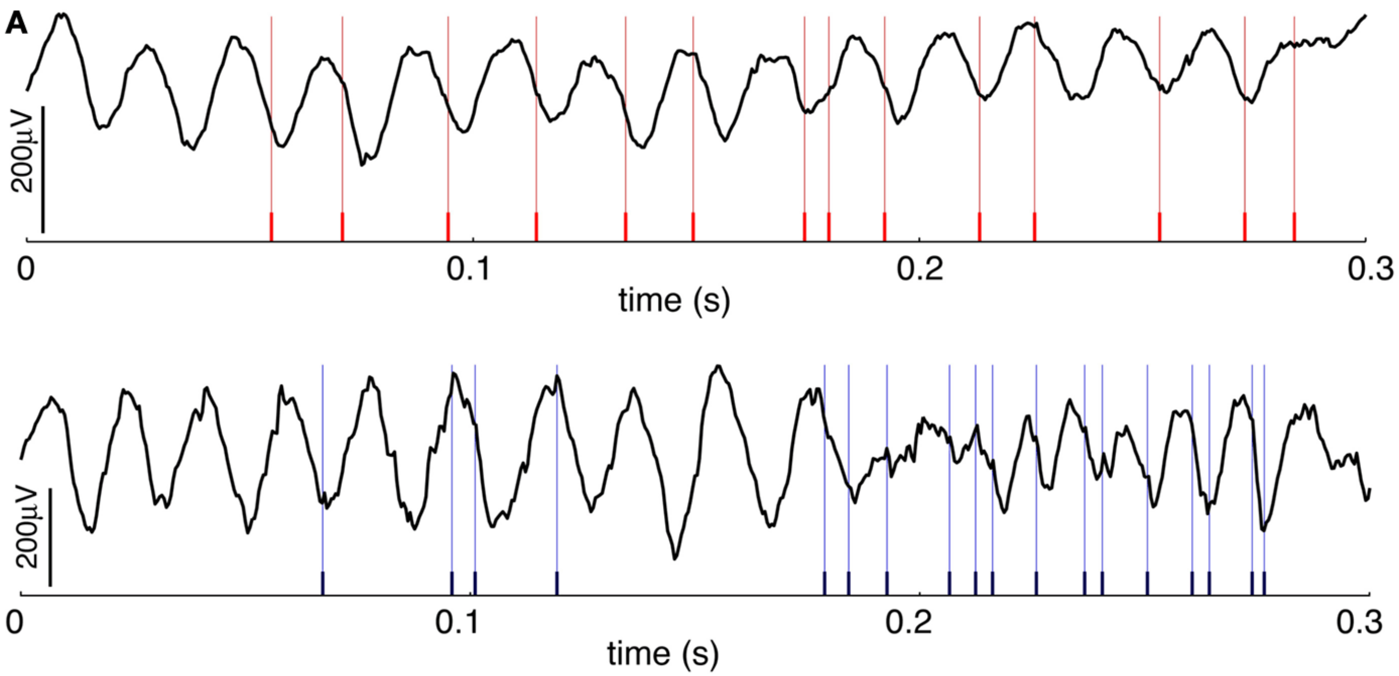
Whiteboard!

- rate estimation by convolution
- replace spike by smooth function

From Spikes to Rate(-estimation)s



Spikes and Local Field Potentials (LFPs)



van der Meer MAA and Redish AD (2009). Low and high gamma oscillations in rat ventral striatum have distinct relationships to behavior, reward, and spiking activity on a learned spatial decision task. Front. Integr. Neurosci. 3:9. doi: 10.3389/neuro.07.009.2009

Whiteboard!

Probability distributions: An overview...

Poisson:

$$P_\lambda(k) = \frac{\lambda^k}{k!}\exp(-\lambda)$$

n -> Inf, p << 1

λ = np

Binomial (Bernoulli process):

$$B_{n,p}(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

λ >> 1

μ=λ, σ^2=λ

np(1-p) > 9

m=np, σ^2=np(1-p)

Gaussian:

$$\rho_{\mu,\sigma}(x) = \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

...in Statistics & Data Analysis lecture!

Expectation values, $E[Z] = \Sigma_i Z_i \, p(Z_i)$ – or – $E[Z] = \int Z \, \rho(Z) \, dZ$

Moments, $\mu_k = E[\,(X-\mu)^k\,]$: Characterize shape of a distribution

Signal-to-Noise ratio, $SNR = \mu/\sigma$: How good is a signal?

Coefficient of Variation, $CV = \sigma/\mu$: variance measure (dimensionless, exponential distribution = 1 = 'normal' variance)

Fano-Factor, $FF = \sigma^2/\mu$: variance measure (Poisson dist.: $FF = 1$)

...here: for uncovering structure within and among spike trains!

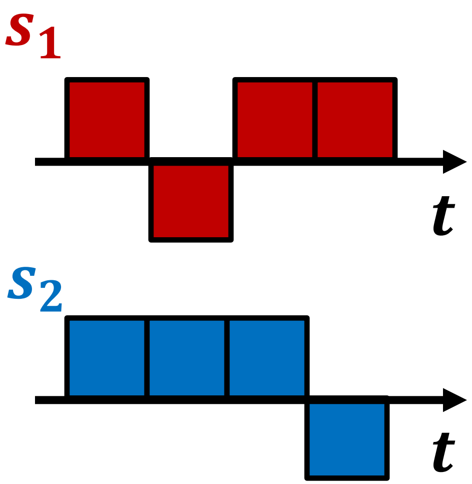
(Cross-)Covariance, $COV(\tau) = E[\,(X_t-\mu_x)\,(Y_{t+\tau}-\mu_y)\,]$: how does a signal vary in time w.r.t another, delayed signal?

(Cross-)Correlation, $COR(\tau) = COV(\tau)\,/(\sigma_x\,\sigma_y)$: covariance normalized, $-1 < COR < 1$

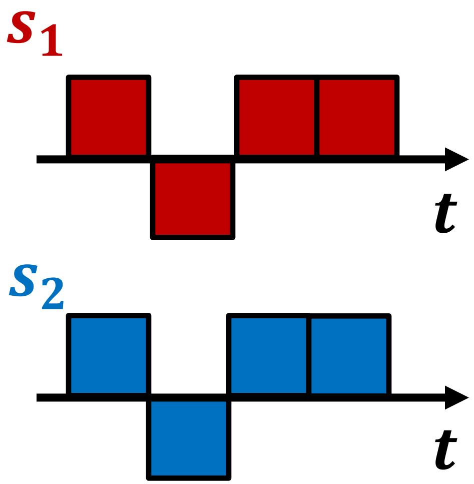
Auto-Covariance, Auto-Correlation, $Y = X$

Correlations: Discovering statistical dependencies between time series

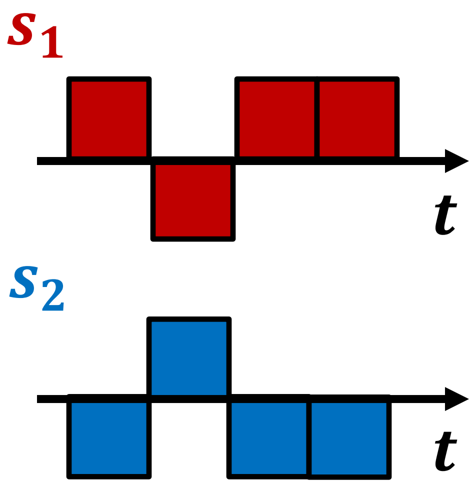
$$c(\tau) \approx \sum_t s_1(t-\tau)s_2(t)$$



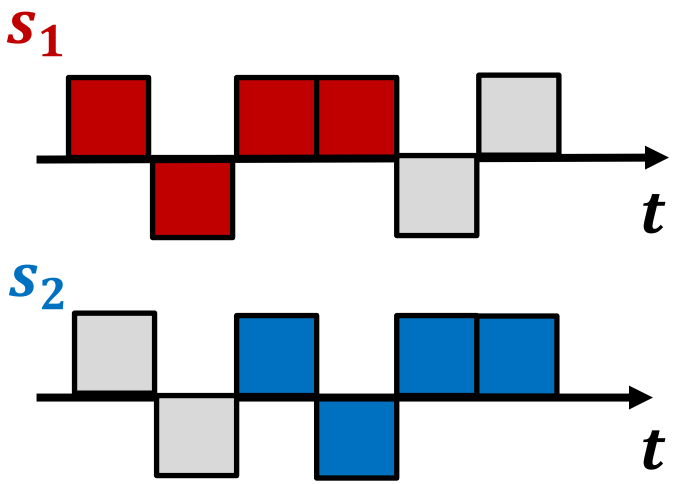
c(0) = 0,
not correlated



c(0) = 4,
correlated



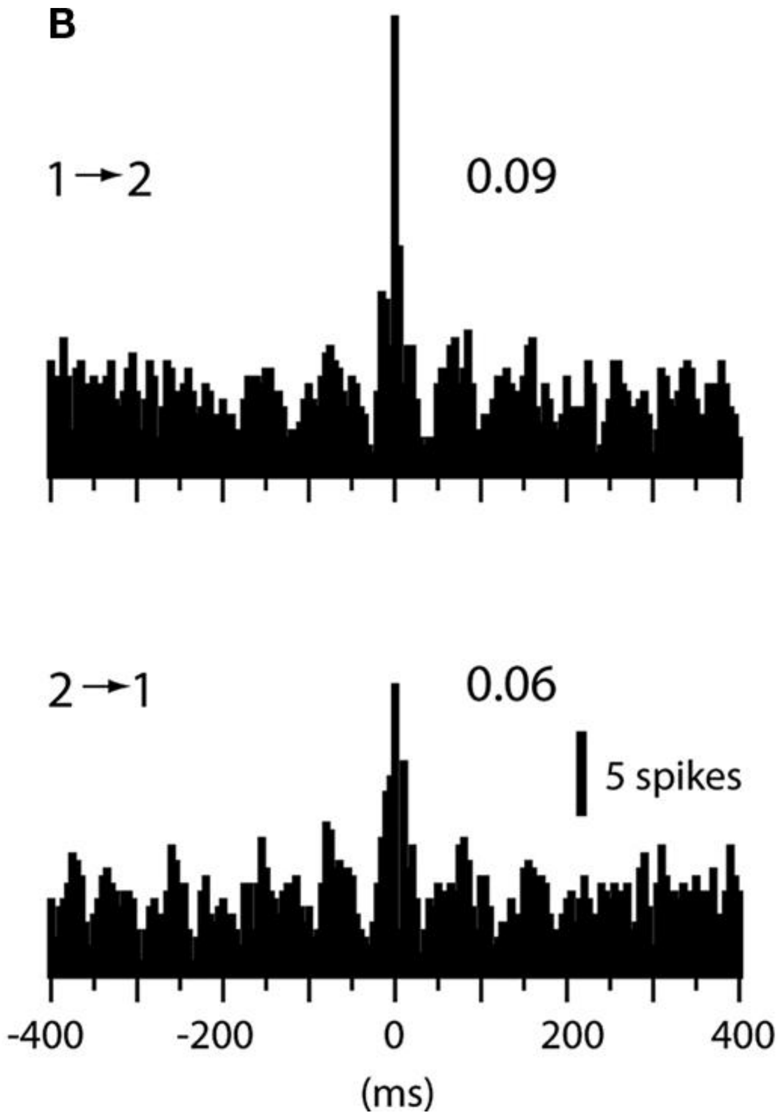
c(0) = -4,
anti-correlated



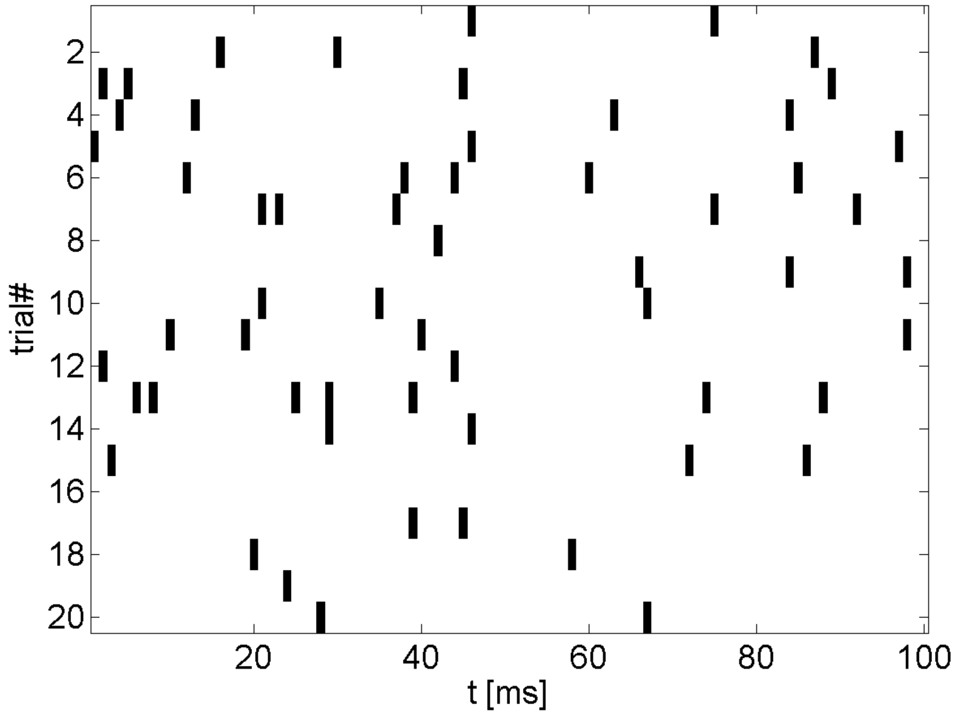
c(2) = 4,
correlated with
time lag 2 in signal 1

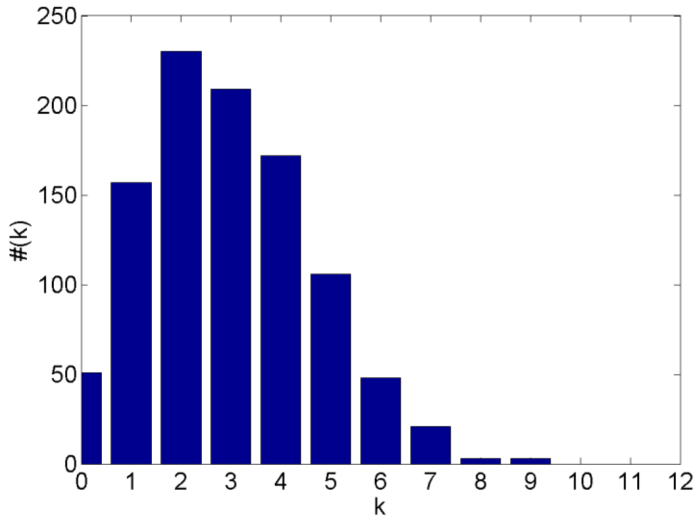
...correlations are in addition **normalized** to **number of observations** and **signal variability**!

Example: Cross-correlograms between spike trains of Purkinje cells



What‘s behind this data? – example I

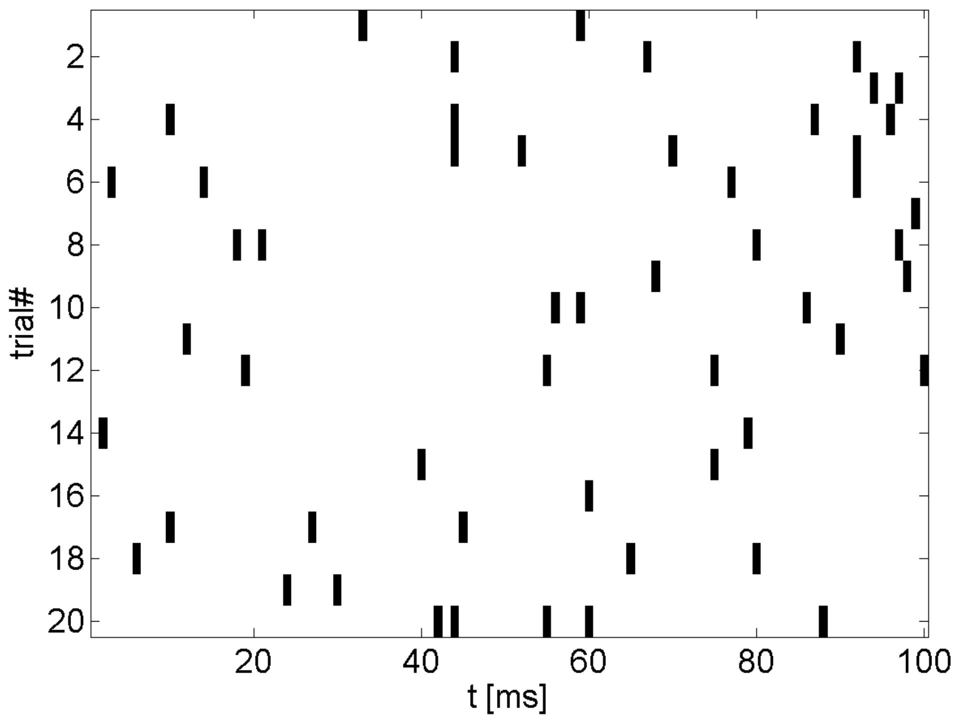


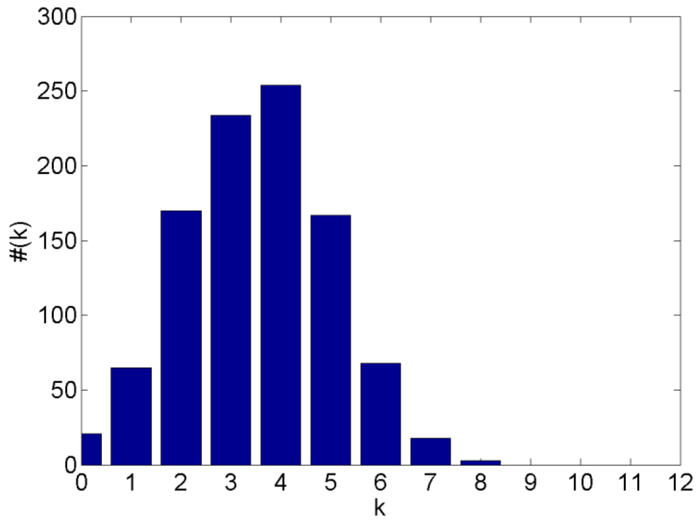


FF=0.97,
CV=0.57,
SNR=1.74

...very likely, it‘s Poisson...

What‘s behind this data? – example II

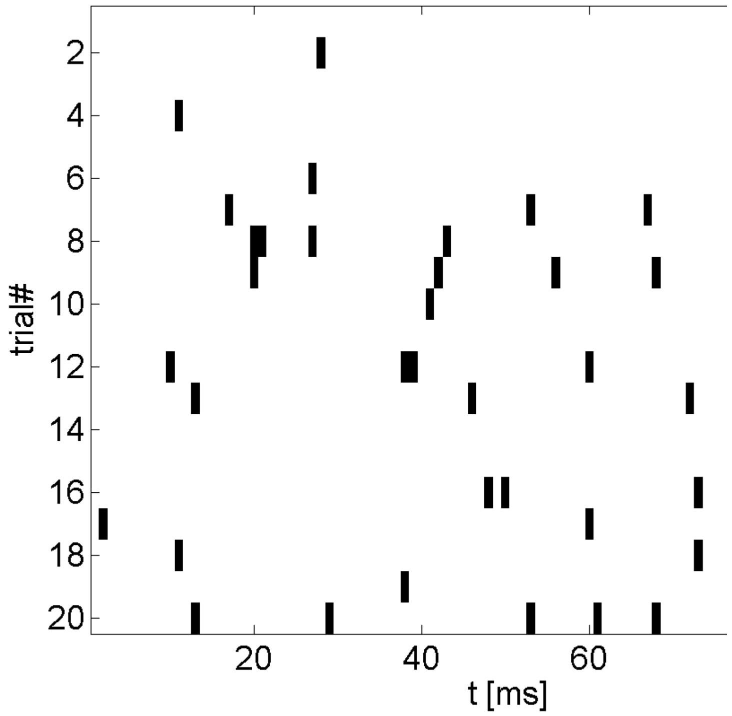


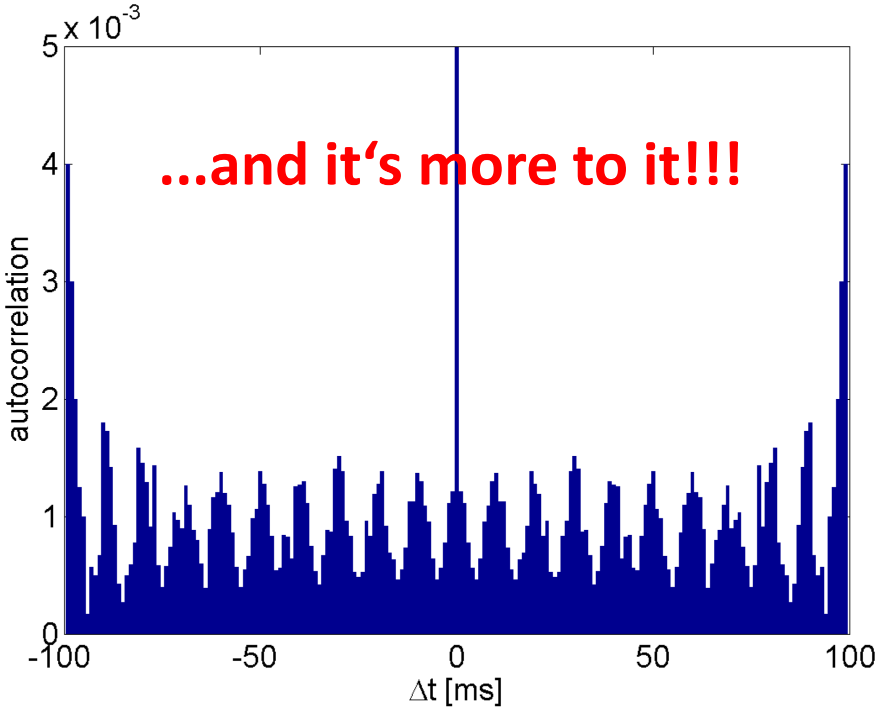


FF=0.63,
CV=0.43,
SNR=2.32

That‘s never Poisson (in fact, it was Gauss...)

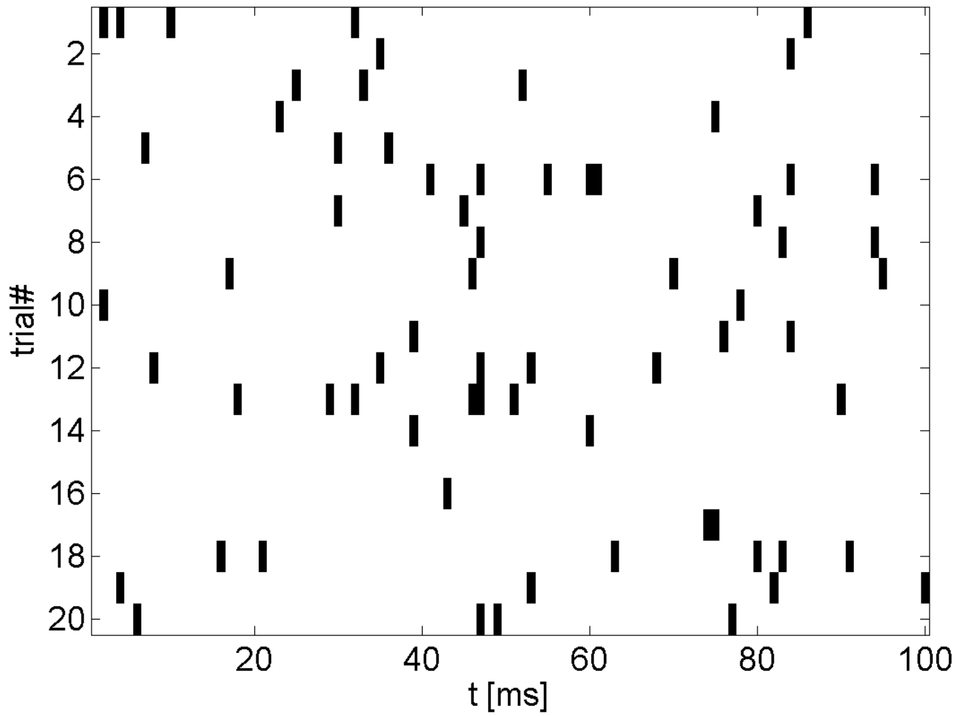
What‘s behind this data? – example III

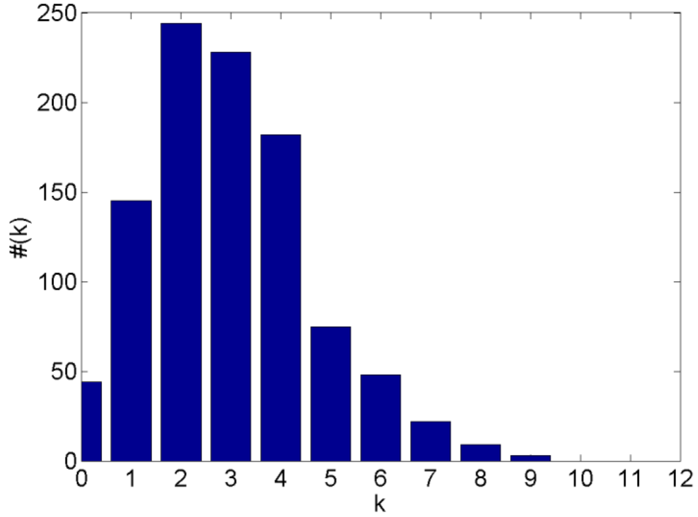




...hey, it‘s Poisson again...

What's behind this data? – example IV





FF=0.95,

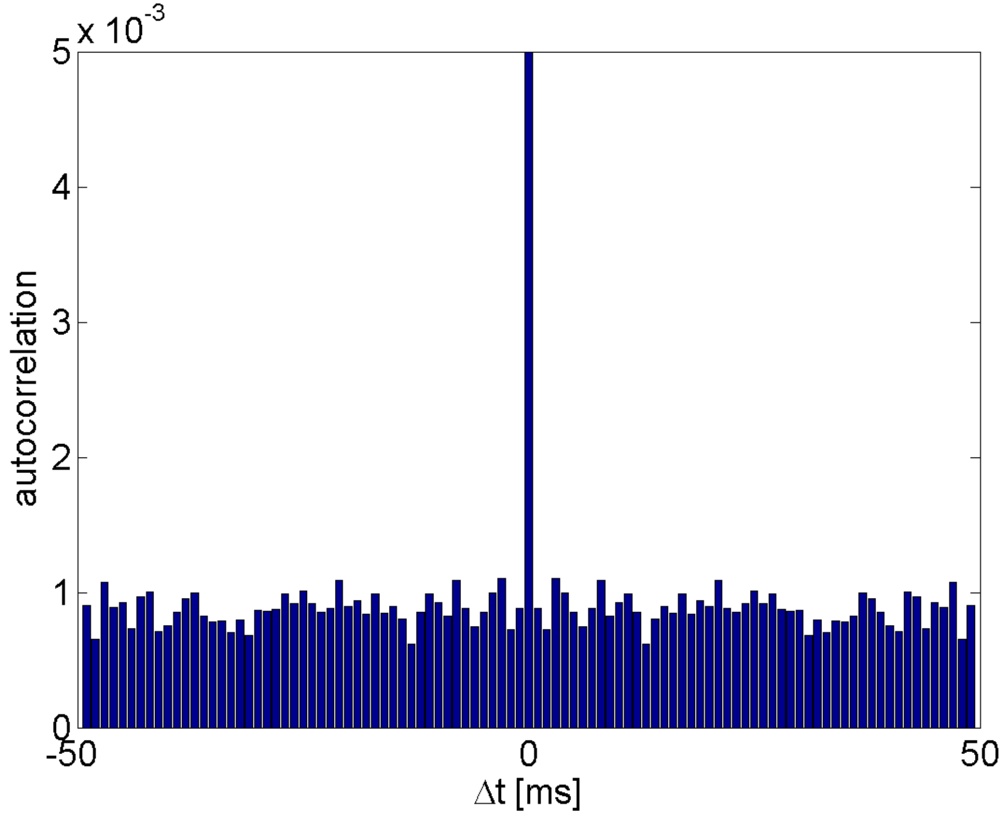
CCV=0.57,

SNR=1.77

...oh, no, not Poisson again...

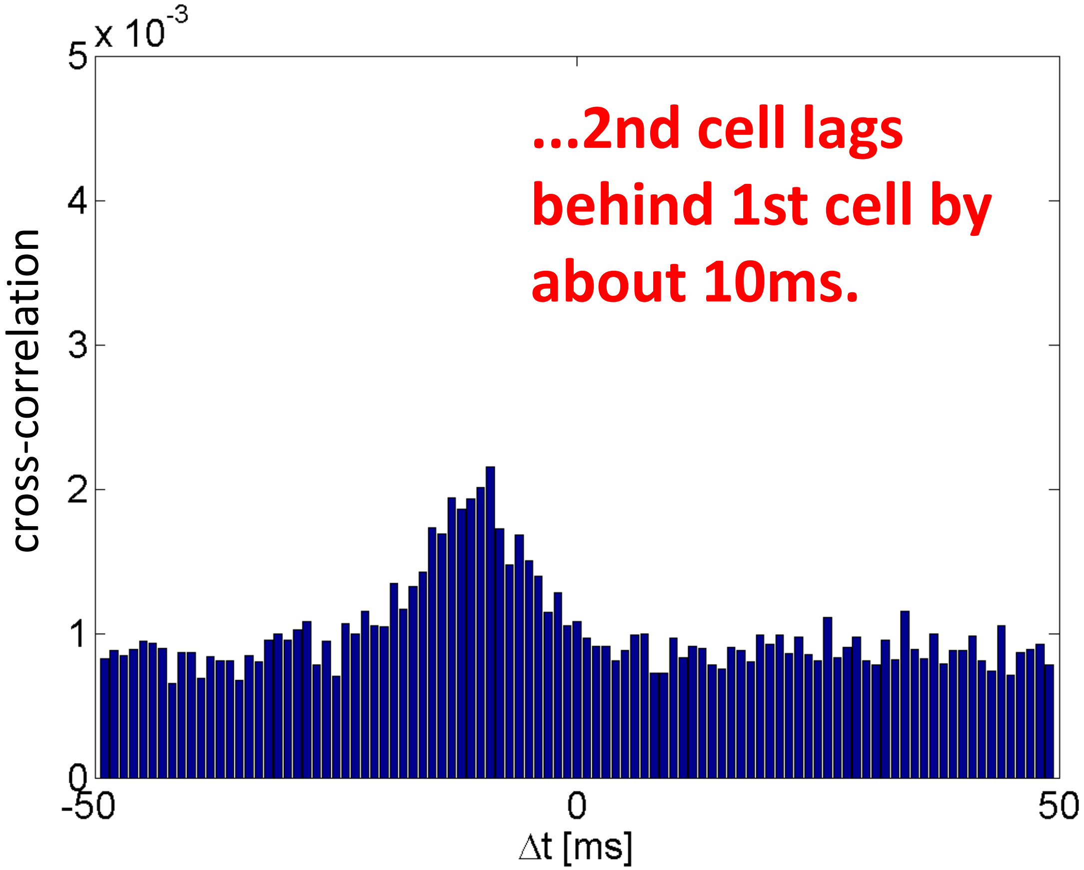
What‘s behind this data? – example IV

...yes, but maybe it‘s oscillatory?



What's behind this data? – example IV

...alas, a second cell has been recorded...



Insights today...

1. The **delta-function** - a nice tool to describe spikes and spike trains
2. Rates and rate estimation by **convolution**!
3. Spike variability: **Bernoulli, Poisson, Gauss and Exponential** distributions
4. Descriptive spike statistics: What's inside a neural recording?