ALTA
Institute for Algebra, Geometry,
Topology and their Applications

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Discrete Mathematics
WS 2025/26

Exercise Set 4

Exercise 4.1 Let n be an arbitrary positive integer, and set $m := \lceil \frac{n}{2} \rceil$. Show that we have the following inequalities:

$$m^{n-m} \leq B_n \leq n!. \tag{1}$$

Investigate for which n the bounds in (1) are strict.

(6 Punkte)

Exercise 4.2 Show that for all positive integers n the following inequalities hold:

$$n! \leq \hat{B}_n \leq n^n. \tag{2}$$

Investigate for which values of n the bounds (2) are strict.

(6 Punkte)

Exercise 4.3 For all nonnegative integers n, prove the following formula:

$$B_n = \sum_{k=0}^{n} \binom{n}{k} (-1)^{n-k} B_{k+1}. \tag{3}$$

(6 Punkte)

Exercise 4.4 Show that, the parity of the Bell numbers is described by the following statement: for all $n \geq 0$, we have

the Bell number B_n is $\begin{cases} \text{even}, & \text{if } n = 3t + 2 \text{ for } t \in \mathbb{Z}_{\geq 0}; \\ \text{odd}, & \text{otherwise.} \end{cases}$

(6 Punkte)

Submission of the exercises: Tues, 11.11.25, before the tutorial (until 12:15) into the postbox 54 in MZH 1st floor, or submission at the beginning of the 12:30-tutorial.