

Exercise Set 5

Exercise 5.1 Find the sum of all the numbers between 1 and 1000 which are products of primes larger than 5.

(6 Punkte)

Exercise 5.2 Show the following formulas by means of a combinatorial argument:

$$\tilde{s}(n, n-2) = \frac{3n-1}{4} \binom{n}{3}, \text{ for } n \geq 3, \quad (1)$$

$$\tilde{s}(n, n-3) = \binom{n}{2} \binom{n}{4}, \text{ for } n \geq 4, \quad (2)$$

(3 Punkte + 3 Punkte)

Exercise 5.3 Show that for all nonnegative integers n , the Catalan number C_n is odd if and only if $n+1$ is a power of 2.

(6 Punkte)

Exercise 5.4 Let $n \geq 1$. A permutation $\pi = \pi_1 \dots \pi_n$ of the set $[n]$ is called *213-avoiding* if there are no $1 \leq i < j < k \leq n$ such that $\pi_k > \pi_i > \pi_j$.

Show that the Catalan number C_n is equal to the number of 213-avoiding permutations of the set $[n]$.

(6 Punkte)

Abgabe der Hausübungen: Tues, 25.11.25, before the tutorial (until 12:15) into the postbox 54 in MZH 1st floor, or submission at the beginning of the 12:30-tutorial.