

Exercise Set 6

Exercise 6.1 Find the inverse (polynomial) of $1 + x + x^2$. (6 Punkte)

Exercise 6.2 Use the method of generating functions for finding an explicit formula for the sequence $\{A_n\}_{n \geq 0}$ given by the recursive data

$$\begin{aligned} A_0 &= A_1 = 1 \\ A_n &= -A_{n-1} - A_{n-2}, \text{ for } n \geq 2. \end{aligned}$$

(6 Punkte)

Exercise 6.3 (a) Find an explicit formula for the so-called Lucas numbers $\{L_n\}_{n \geq 0}$, which are defined by

$$\begin{aligned} L_0 &= 2, \quad L_1 = 1, \\ L_{n+2} &= L_{n+1} + L_n, \text{ for } n \geq 0. \end{aligned}$$

So, the sequence starts 2, 1, 3, 4, 7, 11, 18, 29, 47, (3 Punkte)

(b) Show that for the Fibonacci numbers $\{F_n\}_{n \geq 0}$, F_{2n-1} is divisible by F_{n-1} , for all $n \geq 1$, and the quotient F_{2n-1}/F_{n-1} is precisely the Lucas number L_n . (3 Punkte)

Exercise 6.4 The sequence $\{u_n\}_{n \geq 0}$ satisfies the recursion

$$u_{n+2} = 2u_{n+1} - u_n, \text{ for all } n \geq 0.$$

Find the initial value u_1 , if we know that $u_0 = 1$, and $u_{100} = 101$.

(6 Punkte)

Submission of the exercises: Tues, 02.12.25, before the tutorial (until 12:15) into the postbox 54 in MZH 1st floor, or submission at the beginning of the 12:30-tutorial.