Robot Perception Pt. II

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2 Camera calibration

Image processing

Object recognition

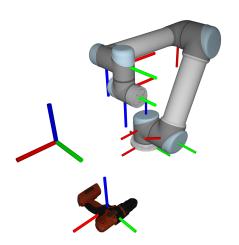
2 Camera calibration

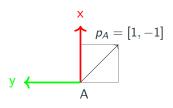
Image processing

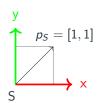
Object recognition

Reference frames

Why do we need reference frames?







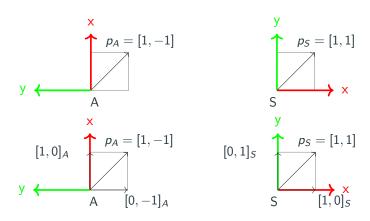
We want to build a machine (function) that takes a point in A coordinates and outputs same point in S coordinates.

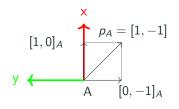
We call this machine:

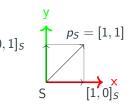
$$R_{SA}$$

And it operates like this:

$$p_S = R_{SA}(p_A)$$







We want to find the function that

$$p_S = R_{SA}(p_A)$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}_{S} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}_{SA} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{A}$$
$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}_{S} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}_{SA} \begin{bmatrix} 0 \\ -1 \end{bmatrix}_{A}$$

()

Let's do the first point first

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}_{S} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}_{SA} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{A}$$
$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}_{S} = 1 \begin{bmatrix} r_{11} \\ r_{21} \end{bmatrix} + 0 \begin{bmatrix} r_{12} \\ r_{22} \end{bmatrix}$$
$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}_{S} = \begin{bmatrix} r_{11} \\ r_{21} \end{bmatrix}$$

Thus

$$r_{11} = 0$$
$$r_{21} = 1$$

Let's do now the second point

$$\begin{bmatrix} 1\\0 \end{bmatrix}_{S} = \begin{bmatrix} r_{11} & r_{12}\\r_{21} & r_{22} \end{bmatrix}_{SA} \begin{bmatrix} 0\\-1 \end{bmatrix}_{A}$$
$$\begin{bmatrix} 1\\0 \end{bmatrix}_{S} = 0 \begin{bmatrix} r_{11}\\r_{21} \end{bmatrix} - 1 \begin{bmatrix} r_{12}\\r_{22} \end{bmatrix}$$
$$\begin{bmatrix} 1\\0 \end{bmatrix}_{S} = -1 \begin{bmatrix} r_{12}\\r_{22} \end{bmatrix}$$

Thus

$$r_{12} = -1$$

 $r_{22} = 0$

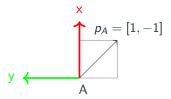
Putting all $r_{11} = 0$, $r_{21} = 1$, $r_{12} = -1$, $r_{22} = 0$ we have:

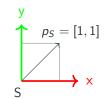
$$R_{SA} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

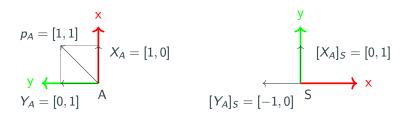
Running now our (unit) test

$$\begin{bmatrix} x_S \\ y_S \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}_{SA} \begin{bmatrix} 1 \\ -1 \end{bmatrix}_A$$

By matrix multiplication $x_S = 1$ and $y_S = 1$. Which is $p_S = [1, 1]$







From what we learned we can build matrices quicker

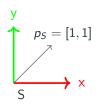
$$\begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}_{SA} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_A = \begin{bmatrix} r_{11} \\ r_{21} \end{bmatrix} := [X_A]_S \qquad \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}_{SA} \begin{bmatrix} 0 \\ 1 \end{bmatrix}_A = \begin{bmatrix} r_{12} \\ r_{22} \end{bmatrix} := [Y_A]_S$$

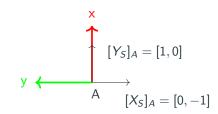
$$R_{SA} = \begin{bmatrix} \uparrow & \uparrow \\ [X_A]_S & [Y_A]_S \\ \downarrow & \downarrow \end{bmatrix}$$

$$R_{SA} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Send unit impulses in A and seeing how they project in S

We have built R_{SA} . Now let's build R_{AS}





$$R_{AS} = \begin{bmatrix} \uparrow & \uparrow \\ [X_S]_A & [Y_S]_A \\ \downarrow & \downarrow \end{bmatrix}$$

$$R_{AS} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Let's look at both matrices

$$R_{SA} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$R_{AS} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

We observe that

$$R_{SA} = R_{AS}^T$$

Or the same as

$$R_{AS} = R_{SA}^T$$

Moreover we see that

$$R_{SA}R_{AS} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} := I$$

Why?

We have that

$$R_{SA} = \begin{bmatrix} \uparrow & \uparrow \\ [X_A]_S & [Y_A]_S \\ \downarrow & \downarrow \end{bmatrix}$$

Thus

$$R_{AS}R_{SA} = R_{SA}^T R_{SA} = \begin{bmatrix} \leftarrow [X_A]_S \to \\ \leftarrow [Y_A]_S \to \end{bmatrix} \begin{bmatrix} \uparrow & \uparrow \\ [X_A]_S & [Y_A]_S \end{bmatrix}$$

Which is

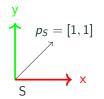
$$R_{SA}^T R_{SA} = \begin{bmatrix} [X_A]_S \cdot [X_A]_S & [X_A]_S \cdot [Y_A]_S \\ [Y_A]_S \cdot [Y_A]_S & [Y_A]_S \cdot [Y_A]_S \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

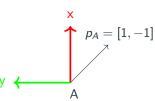
Where \cdot is the dot product and we use these (easy to prove) facts:

- The dot product of a normalized vector with itself is 1.
- The dot product of two orthogonal vectors is 0.

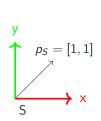
Passive vs. Active

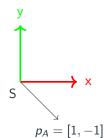
Passive: Space remains the same but the frame of reference changes.



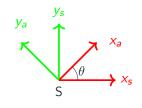


Active: Frame of reference remains the same but space changes.



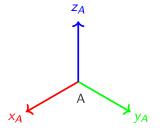


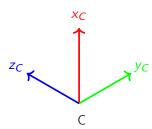
Generic rotation matrix for 2D



$$R_{SA} = \begin{bmatrix} \uparrow & \uparrow \\ [X_A]_S & [Y_A]_S \\ \downarrow & \downarrow \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Extension to 3D





Let's build R_{AC} :

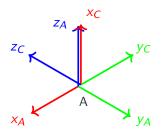
$$R_{AC} = \begin{bmatrix} \uparrow & \uparrow & \uparrow \\ [X_C]_A & [Y_C]_A & [Z_C]_A \\ \downarrow & \downarrow & \downarrow \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

Making our test

$$\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Extension to translations

We have only rotated frames, and computed coordinates in rotated frames.

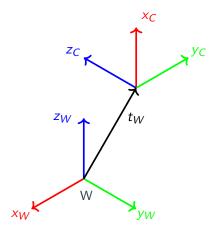


But we can also translate frames.

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} & tx \\ r_{21} & r_{22} & r_{23} & ty \\ r_{31} & r_{31} & r_{33} & tz \\ 0 & 0 & 0 & 1 \end{bmatrix}_{CA}$$

Extension to translations

What is $t = [t_x, t_y, t_z]$?

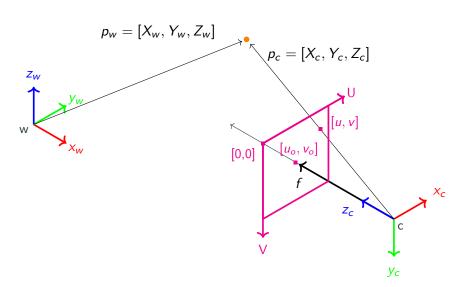


$$\begin{bmatrix} R_{CW} & R_{CW}(-t_W) \\ \mathbf{0}_{3\times 1} & 1 \end{bmatrix}_{CW}$$

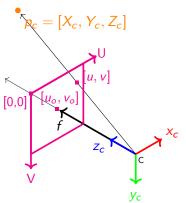
Camera calibration

Image processing

Object recognition



From our previous lecture we know:



The pinhole camera model projects p_c to the image plane using:

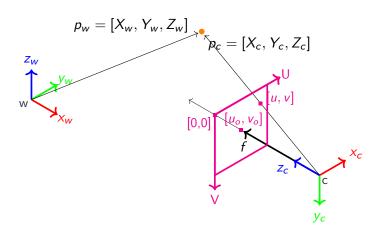
$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda u \\ \lambda v \\ \lambda \end{bmatrix} = \begin{bmatrix} k_u f & 0 & u_0 \\ 0 & k_v f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = K \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda u \\ \lambda v \\ \lambda \end{bmatrix} = \begin{bmatrix} k_u f & 0 & u_0 \\ 0 & k_v f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = K \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}$$

To use homogenous points ([$X_c, Y_c, Z_c, 1$]) we extend K with a 0's column

$$[K|0] = \begin{bmatrix} k_u f & 0 & u_0 & 0 \\ 0 & k_v f & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = [K|0] = \begin{bmatrix} k_u f & 0 & u_0 & 0 \\ 0 & k_v f & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$



From what we know now of affine transformations we can easily compute

$$A_{cw} = \begin{bmatrix} R_{cw} & T_{cw} \\ \mathbf{0} & 1 \end{bmatrix} \tag{2}$$

Putting all together we have intrinsics and extrinsics camera parameters

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} k_u f & 0 & u_0 & 0 \\ 0 & k_v f & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & tx \\ r_{21} & r_{22} & r_{23} & ty \\ r_{31} & r_{31} & r_{33} & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = [K_{\text{intrinsic}}|0]M_{\text{extrinsic}}$$

How do we obtain the internal and external parameters?

The main steps are:

- Obtain pairs of $[X_w, Y_w, Z_w]$ and [u, v].
- Minimize the error between pairs.

There are multiple algorithms and openCV has an out-of-the-box solution.

How does it look in openCV?

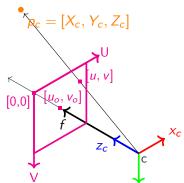


Figure: Calibration pattern

Correspondence between a known object dimensions in 3D space and easy to located coordinates in 2D image space.

Applications of camera calibration:

- Distance measurement
- Pose estimation
- Drawing
- Image rectification



2 Camera calibration

Image processing

Object recognition

From our previous lecture:

- Higher-level image processing might require binary images
 - We can apply a threshold operation
- Per-color threshold operations can help us create classifiers.
 - "How much red does our image has?"



Threshold = 160 Replace pixels below by 0 (black), keep pixels above.



We can also apply convolution operations to images

Convolutions apply the same operation in image patches

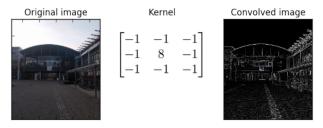


Figure: Convolution operation for image processing.

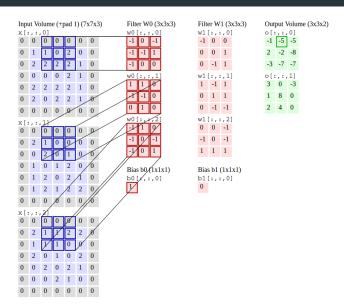


Figure: Extended convolution operation

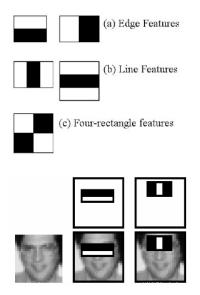


Figure: Haar features

Hough transform

If we we have a data point (x_i, y_i) and a linear model:

$$y_i = mx_i + b$$

A point can be explained by an infinite amount of lines $(m,b) \in R \times R$

$$b = -x_i m + y_i$$

Vote in parameter space using an accumulator function.

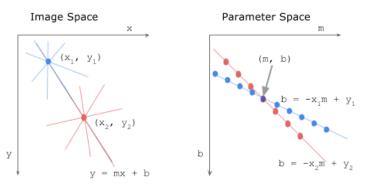


Figure: Hough space (source)

Hough transform

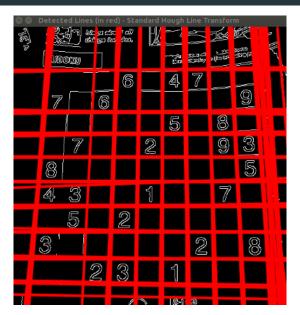


Figure: Application of Hough transform

Hough transform

If we we have a data point (x_i, y_i) and a circle model:

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

A point can be explained as circles $(a, b) \in R \times R$

$$(a-x_i)^2+(b-y_i)^2=r^2$$

Vote in parameter space using an accumulator function.

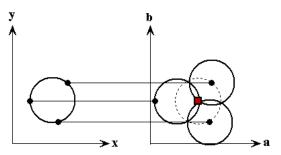


Figure: Hough space (source)

Camera calibration

Image processing

4 Object recognition

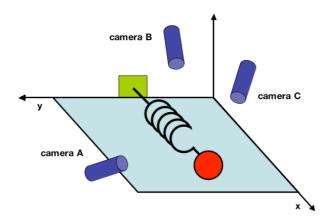


Figure: Experiment setup [Shlens 2014]

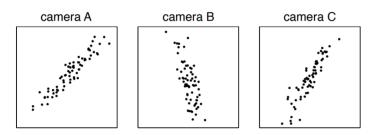


Figure: Experiment output [Shlens 2014]

Naive base coordinate description of a single sample

$$X = \begin{bmatrix} x_A \\ y_A \\ x_B \\ y_B \\ x_C \\ y_C \end{bmatrix}$$

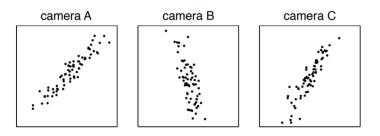


Figure: Experiment output [Shlens 2014]

However we have multiple samples:

$$\mathbf{X} = \begin{bmatrix} x_A^1 & x_A^2 & \dots & x_A^n \\ y_A^1 & y_A^2 & \dots & y_A^n \\ x_B^1 & x_B^2 & \dots & x_B^n \\ y_B^1 & y_B^2 & \dots & y_B^n \\ x_C^1 & x_C^2 & \dots & x_C^n \\ y_C^1 & y_C^2 & \dots & y_C^n \end{bmatrix}$$

We know that we only need one dimension.

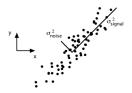


Figure: Signal to noise [Shlens 2014]

Moreover orthogonality can represent noise.

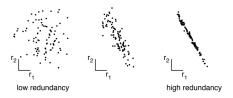


Figure: Redudancy [Shlens 2014]

We can transform our data X

$$\mathbf{X} = \begin{bmatrix} x_A^1 & x_A^2 & \dots & x_A^n \\ y_A^1 & y_A^2 & \dots & y_A^n \\ x_B^1 & x_B^2 & \dots & x_B^n \\ y_B^1 & y_B^2 & \dots & y_B^n \\ x_C^1 & x_C^2 & \dots & x_C^n \\ y_C^1 & y_C^2 & \dots & y_C^n \end{bmatrix}$$

We can compute the redundancy (linear correlation) in the following way:

$$C = \frac{1}{n} \mathbf{X} \mathbf{X}^T$$

The explicit values of matrix C are as follows:

$$C = \begin{bmatrix} x_A^1 & x_A^2 & \dots & x_A^n \\ y_A^1 & y_A^2 & \dots & y_A^n \\ x_B^1 & x_B^2 & \dots & x_B^n \\ y_B^1 & y_B^2 & \dots & y_B^n \\ x_C^1 & x_C^2 & \dots & x_C^n \\ y_C^1 & y_C^2 & \dots & y_C^n \end{bmatrix} \begin{bmatrix} x_A^1 & y_A^1 & x_B^1 & y_B^1 & x_C^1 & y_C^1 \\ x_A^2 & y_A^2 & x_B^2 & y_B^2 & x_C & y_C^1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_A^n & y_A^n & x_B^n & y_B^n & x_C^1 & y_C^1 \end{bmatrix}$$

$$C = \begin{bmatrix} x_A \cdot x_A & x_A \cdot y_A & x_A \cdot x_B & x_A \cdot y_B & x_A \cdot x_C & x_A \cdot y_C \\ y_A \cdot x_A & y_A \cdot y_A & y_A \cdot x_B & y_A \cdot y_B & y_A \cdot x_C & y_A \cdot y_C \\ y_A \cdot x_A & x_B \cdot y_A & x_B \cdot x_B & x_B \cdot y_B & x_B \cdot x_C & x_B \cdot y_C \\ y_B \cdot x_A & y_B \cdot y_A & y_B \cdot x_B & y_B \cdot y_B & y_B \cdot x_C & y_B \cdot y_C \\ x_C \cdot x_A & x_C \cdot y_A & x_C \cdot x_B & x_C \cdot y_B & x_C \cdot x_C & x_C \cdot y_C \\ y_C \cdot x_A & y_C \cdot y_A & y_C \cdot x_B & y_C \cdot y_B & y_C \cdot x_C & y_C \cdot y_C \end{bmatrix}$$

We want to find a base in which all diagonal are maximized.

$$\sigma = \frac{\sum_{i} (x_i - \mu_x)^2}{n} \tag{3}$$

and all off diagonal elements are zero (how much x tells me about y).

$$cov(x,y) = \sum_{i} \frac{(x_i - \mu_x)(y_i - \mu_y)}{n}$$
 (4)



High correlation with x and y, imply I can discard one and use only the other.

We want a new basis where:

- Point to maximum variance (look where there is more change).
- Minimize linear correlation between values (remove redundancy).
- Basis vectors are orthogonal

In other words we want to diagonalize C.

Since the diagonal elements are variance and off-diagonal are covariance.

So we want a matrix P such that \hat{C} is diagonal (P is a change of basis)

$$\hat{C} = \frac{1}{n} (PX)(PX)^T$$

$$\hat{C} = \frac{1}{n} (P(XX^T)P^T)$$

$$\hat{C} = P(\frac{1}{n}XX^T)P^T$$

$$\hat{C} = PCP^T$$

If we choose P to be equal to the eigenvectors of C i.e. E

$$P = E$$

Then \hat{C} will be a diagonal matrix. Eigenvectors satisfy

$$PE_i = \lambda E_i$$

$$\hat{C} = PCP^{\mathsf{T}}$$

 $CE_i = \lambda_i E_i$

Assume

If we multiply by a basis vector
$$e_i = [0, \dots, 1, \dots, 0]$$

$$\hat{C}e_i = PCP^Te_i$$

 $P^T e_i = F_i$

 $\hat{C}e_i = PCE_i = P\lambda_i E_i$

$$P = \left| \begin{array}{c} \vdots \\ \leftarrow E_n \end{array} \right|$$

$$P = \begin{vmatrix} \leftarrow E_1 \rightarrow \\ \vdots \\ \leftarrow E_n \rightarrow \end{vmatrix}$$

$$\left[\begin{smallmatrix} -1 \\ -1 \end{smallmatrix}\right]$$

(7)

(9)

(10)

From our eigenvector assumption

$$\hat{C}e_i = P\lambda_i E_i \tag{11}$$

$$\hat{C}e_i = \lambda_i P E_i \tag{12}$$

From our previous assumption in equation 9 we have $e_i = PE_i$ thus:

$$\hat{C}e_i = \lambda_i e_i \tag{13}$$

Thus $\hat{C}e_i$ is a diagonal matrix (since all values not i are zero)

Mea culpa: If C is orthogonal then is orthogonally diagonziable i.e $P^TP = I$. In other words the vectors E_i are orthogonal

So the final result is that if we want diagonalize C

$$C = \begin{bmatrix} x_{A} \cdot x_{A} & x_{A} \cdot y_{A} & x_{A} \cdot x_{B} & x_{A} \cdot y_{B} & x_{A} \cdot x_{C} & x_{A} \cdot y_{C} \\ y_{A} \cdot x_{A} & y_{A} \cdot y_{A} & y_{A} \cdot x_{B} & y_{A} \cdot y_{B} & y_{A} \cdot x_{C} & y_{A} \cdot y_{C} \\ x_{B} \cdot x_{A} & x_{B} \cdot y_{A} & x_{B} \cdot x_{B} & x_{B} \cdot y_{B} & x_{B} \cdot x_{C} & x_{B} \cdot y_{C} \\ y_{B} \cdot x_{A} & y_{B} \cdot y_{A} & y_{B} \cdot x_{B} & y_{B} \cdot y_{B} & y_{B} \cdot x_{C} & y_{B} \cdot y_{C} \\ x_{C} \cdot x_{A} & x_{C} \cdot y_{A} & x_{C} \cdot x_{B} & x_{C} \cdot y_{B} & x_{C} \cdot x_{C} & x_{C} \cdot y_{C} \\ y_{C} \cdot x_{A} & y_{C} \cdot y_{A} & y_{C} \cdot x_{B} & y_{C} \cdot y_{B} & y_{C} \cdot x_{C} & y_{C} \cdot y_{C} \end{bmatrix}$$

We should choose the basis vectors E_i to describe our new base

$$P = \begin{bmatrix} \leftarrow E_1 \to \\ \vdots \\ \leftarrow E_n \to \end{bmatrix} \tag{14}$$

Principal component analysis (PCA) algorithm:

- Get data
- Compute data mean
- Subtract mean from data
- Compute covariance matrix (C).
- Find base the minimizes correlation, looks for maximum variance.
- Remove base vectors with small variance
- Project data to new base

What is a vector?

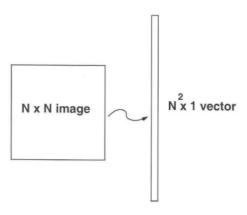


Figure: Image to vector

Get a large amount of faces and compute the mean face

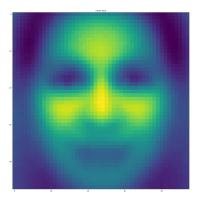
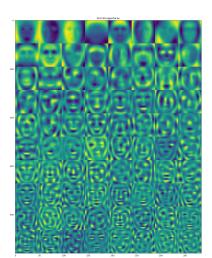
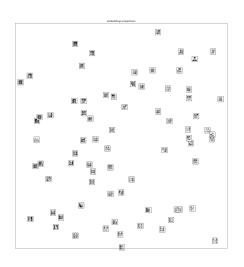


Figure: Mean face

Subtract mean face from all faces and compute eigenvectors (eigenfaces) of the covariance matrix.



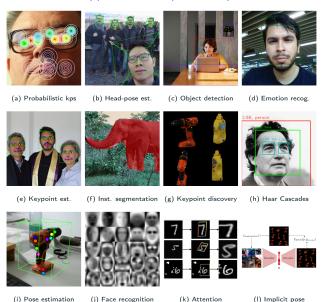
Now project any face to the eigenspace



Additional material

- Computer Vision: Algorithms and Applications, 2nd ed.
- First principles of computer vision
- Deep Learning with Python

https://github.com/oarriaga/paz



Examples implemented in PAZ

Figure: PAZ examples