Standardverteilung: $\mu = 0$ and $\sigma^2 = 1$
Wölbung:

$$\mathbb{E}[X^4] = \int_{-\infty}^{\infty} \frac{x^4}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

$$\frac{d(x^3 e^{\frac{-x^2}{2}})}{dx} = 3x^2 e^{\frac{-x^2}{2}} - x^4 e^{\frac{-x^2}{2}}$$

$$\begin{aligned}
\mathbb{E}[X^4] &= \int_{-\infty}^{\infty} \frac{x^4}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \\
&= \int_{-\infty}^{\infty} \frac{3x^2}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}\, dx + \int_{-\infty}^{\infty} \frac{x^4 - 3x^2}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}\, dx \\
&= 3\int_{-\infty}^{\infty} \frac{x^2}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}\, dx + \left[\frac{-x^3}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}\right]_{-\infty}^{\infty} \\
&= 3\,\mathbb{E}[X^2] + 0 \\
&= 3
\end{aligned}$$